OCR
Oxford Cambridge and RSA

Thursday 6 June 2019 – Afternoon

A Level Further Mathematics A

Y541/01 Pure Core 2

Time allowed: 1 hour 30 minutes

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You must have:
- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

1 **In this question you must show detailed reasoning.**

(a) By using partial fractions show that $\sum_{r=1}^{n} \frac{1}{r^2+3r+2} = \frac{1}{2} - \frac{1}{n+2}$. **[5]**

(b) Hence determine the value of $\sum_{r=1}^{\infty} \frac{1}{r^2+3r+2}$. **[2]**

2 **(a)** A plane Π has the equation $\mathbf{r}.\begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix} = 15$. C is the point (4, −5, 1).

Find the shortest distance between Π and C. **[3]**

(b) Lines l_1 and l_2 have the following equations.

$$l_1: \mathbf{r} = \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 4 \\ -2 \end{pmatrix}$$

$$l_2: \mathbf{r} = \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

Find, in exact form, the distance between l_1 and l_2. **[5]**

3 **In this question you must show detailed reasoning.**

Show that $\int_5^{\infty} (x-1)^{-\frac{3}{2}} \mathrm{d}x = 1$. **[5]**

4 A 2-D transformation T is a shear which leaves the y-axis invariant and which transforms the object point (2, 1) to the image point (2, 9). **A** is the matrix which represents the transformation T.

(a) Find **A**. **[3]**

(b) By considering the determinant of **A**, explain why the area of a shape is invariant under T. **[2]**

5 A particle of mass 2 kg moves along the x-axis. At time t seconds the velocity of the particle is $v\,\text{ms}^{-1}$.

The particle is subject to two forces.

- One acts in the positive x-direction with magnitude $\frac{1}{2}t\,\text{N}$.
- One acts in the negative x-direction with magnitude $v\,\text{N}$.

(a) Show that the motion of the particle can be modelled by the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} + \frac{1}{2}v = \frac{1}{4}t.$$ **[1]**

The particle is at rest when $t = 0$.

(b) Find v in terms of t. **[5]**

(c) Find the velocity of the particle when $t = 2$. **[1]**

When $t = 2$ the force acting in the **positive** x-direction is replaced by a constant force of magnitude $\frac{1}{2}\text{N}$ in the same direction.

(d) Refine the differential equation given in part **(a)** to model the motion for $t \geqslant 2$. **[1]**

(e) Use the refined model from part **(d)** to find an exact expression for v in terms of t for $t \geqslant 2$. **[3]**

6 A is a fixed point on a smooth horizontal surface. A particle P is initially held at A and released from rest.

It subsequently performs simple harmonic motion in a straight line on the surface. After its release it is next at rest after 0.2 seconds at point B whose displacement is 0.2 m from A. The point M is halfway between A and B.

The displacement of P from M at time t seconds after release is denoted by x m.

(a) On the axes provided in the Printed Answer Booklet, sketch a graph of x against t for $0 \leqslant t \leqslant 0.4$. **[4]**

(b) Find the displacement of P from M at 0.75 seconds after release. **[2]**

7 In an Argand diagram the points representing the numbers $2 + 3\mathrm{i}$ and $1 - \mathrm{i}$ are two adjacent vertices of a square, S.

(a) Find the area of S. **[3]**

(b) Find all the possible pairs of numbers represented by the other two vertices of S. **[4]**

8 **In this question you must show detailed reasoning.**

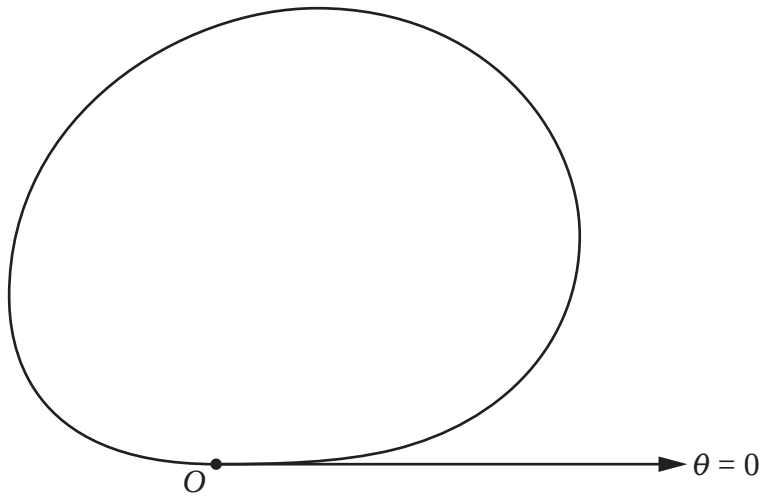
(a) By writing $\sin\theta$ in terms of $\mathrm{e}^{\mathrm{i}\theta}$ and $\mathrm{e}^{-\mathrm{i}\theta}$ show that

$$\sin^6\theta = \tfrac{1}{32}(10 - 15\cos 2\theta + 6\cos 4\theta - \cos 6\theta).$$ **[5]**

(b) Hence show that $\sin\tfrac{1}{8}\pi = \dfrac{1}{2}\sqrt[6]{20 - 14\sqrt{2}}$. **[3]**

9 In this question you must show detailed reasoning.

The diagram below shows the curve $r = \sqrt{\sin\theta}\,e^{\frac{1}{3}\cos\theta}$ for $0 \leqslant \theta \leqslant \pi$.



(a) Find the exact area enclosed by the curve. **[4]**

(b) Show that the greatest value of r on the curve is $\sqrt{\frac{\sqrt{3}}{2}}e^{\frac{1}{6}}$. **[7]**

10 (a) Use differentiation to find the first two non-zero terms of the Maclaurin expansion of $\ln\left(\frac{1}{2}+\cos x\right)$. **[4]**

(b) By considering the root of the equation $\ln\left(\frac{1}{2}+\cos x\right) = 0$ deduce that $\pi \approx 3\sqrt{3\ln\left(\frac{3}{2}\right)}$. **[3]**

END OF QUESTION PAPER

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